SOME PROBLEMS OF THE GENERALIZATION IN DIMENSIONLESS NUMBERS OF THE CHARACTERISTICS OF GAS-STABILIZED ELECTRIC ARCS

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It is shown that the current-voltage characteristics of electric arcs, stabilized in a cylindrical channel by a flow of helium, argon, nitrogen or oxygen, can be generalized by means of a dimensionless criterial equation.

Studies have recently been published [1–3] in which the characteristics of electric arcs are generalized by the methods of similarity theory. In these studies it is shown that, in spite of the complexity of the arc processes, it is possible to confine oneself to a small number of criteria only. In gas-stabilized arcs, for example, the principal role is played by the criterion $Ku = Gd\sigma_0 h_0/I^2$, which characterizes the heat release from the arc column due to convective heat transfer. In generalizing the current-voltage characteristics of certain arc heaters with vortex gas stabilization it is possible to neglect all other criteria. However, in many cases it is necessary to introduce other criteria as well.

The published data relate in each individual case to a certain specific medium in which the arc burns. Thus it is possible to disregard the physical properties of the medium and carry out the generalization in dimensional complexes corresponding to the initial dimensionless criteria [1–3]. This approach makes it possible to reduce the number of independent variables, which considerably simplifies the processing of the experimental data. The number of experiments required is also reduced, since the processes are self-similar with respect to each of the parameters entering into the complex. In particular, this makes it possible to model powerful electric-arc equipment under laboratory conditions. However, the generalization in dimensional complexes is incomplete, since the physical properties are left out. Accordingly, the experimental data cannot be made to apply to heaters in which other stabilizing media are used.

The generalization of the current-voltage characteristics of gas-stabilized electric arcs involves considerable difficulties, mainly due to the need to introduce new criteria reflecting the influence of the physical properties of the gases. Since the arc itself constitutes a complex system of interrelated processes, the total number of criteria is quite large. However, when the number of independent generalized variables is increased, elucidation of the laws involves a considerable expansion of the volume of experimental data. Correspondingly, the reduction of these data becomes more difficult, and the accuracy of the generalized empirical formulas obtained is reduced. Nonetheless, these generalized formulas can be very useful in designing electric-arc apparatus, since they permit a considerable reduction in the volume of preliminary research, which is normally bound up with an appreciable expenditure of time and money.

In converting from characteristics represented in dimensional complexes to dimensionless criterial equations it is necessary, first of all, to find the characteristic values of the physical properties. In vortex-stabilized heaters under conditions of developed turbulence the generalization can be based on a single characteristic number, $I^2/Gdh_0\sigma_0$, which represents the convective heat transfer process. This number contains characteristic values of the electrical conductivity and enthalpy, σ_0 and h_0. If $\sigma = f(h)$ can be represented in the form of a power function $\sigma = \sigma_0(h/h_0)^n$, then as characteristic values it is possible to take the values of the properties at any of the points lying on the approximating curve. In order to take into account the physical properties of the gas, as an additional characteristic number it is necessary to introduce the exponent n. Since the basic process is the heat transfer process, in many cases it may be possible to confine oneself to these two characteristic numbers.

The relation $\sigma = f(h)$ is a complex one. At small h the gas is not ionized, and the conductivity is zero. Then thermal ionization, accompanied by a sharp increase in conductivity, begins. However, this increase slows down, since the increase in electron concentration per unit volume due to ionization of atoms and molecules is compensated by a decrease in gas density. In this case there may be a drop in conductivity, which, in the case of multiple ionization, leads to a further increase.

To approximate such a complex relationship over a wide range of enthalpy variation by means of some simple expression is not possible. However, it is possible to divide the curve into individual segments, each of which is then approximated by a power function. In gas-stabilized arcs the processes are usually confined to the region of single ionization. Therefore, the curve $\sigma = f(h)$ can be divided into three segments. In the first segment $\sigma = 0$, while the other two are approximated by power relations with different exponents.

In Fig. 1 the conductivity has been calculated by the method of [6]. As the figure shows, the first segment, on which the conductivity is zero, depends mainly on the molecular weight of the gas. The lighter the gas, the higher the enthalpies at which the marked increase in conductivity begins. For example, for hydrogen the initial enthalpy is two orders higher than for argon. The conductivity vs. enthalpy curves shown in the figure in logarithmic coordinates can be approximated, with an accuracy good enough for practical purposes, by two line segments.

The current voltage characteristics are determined by the resistance of the arc. Therefore the form of the characteristics depends on the enthalpy of the gas-discharge plasma. The enthalpy, in its turn, depends on the current and the conditions of heat transfer between the arc and the surrounding medium. At low currents of the order of tens of amperes and below, the current-voltage characteristics are usually drooping curves, which testifies to a strong increase in conductivity with rise in temperature. However, at currents of hundreds of amperes and above, the characteristics of stabilized arcs start to increase, while the descending characteristics of unstabilized arcs become less steep. This indicates that the main part of the arc column is located in the region of weak variation of conductivity. Since the currents in powerful industrial apparatus are equal to hundreds and thousands of amperes, we shall base our study on the case of a weak dependence of conductivity on enthalpy. Accordingly, we can employ a rougher approximation, replacing the second segment of steep increase in conductivity by a vertical straight line. This kind of approximation is shown by the broken lines in Fig. 1. As the characteristic quantities σ_0 and h_0 we can take the values corresponding to the points of inflection of the polygonal curve. These values of σ_0 and h_0 are presented in the table, together with the slope of the approximating straight line, for various gases (whose characteristics are shown in Fig. 3).

The conductivity of the plasma depends not only on temperature but also on pressure. Therefore variation of the pressure may lead to variation of the coefficients of the approximating formulas, which to some extent reflects the influence of pressure on the arc characteristics.

To what degree the above method of approximating the physical properties of gases and finding their characteristic values is suitable for generalizing in dimensionless numbers the current-voltage characteristics of electric-arc heaters is discernible from the example of the apparatus with vortex-stabilized arc whose characteristic is shown in Fig. 2.

From the figure it is clear that the arc voltage strongly depends on the kind of gas, in particular, the higher the molecular weight of the gas, the lower the voltage. Apparently, this conclusion does not hold over the entire range of currents, since the steepness of the characteristic also depends on the nature of the gas. The characteristics for inert gases are flatter than those for diatomic gases. It is possible to trace the relationship between the steepness of the $\sigma = f(h)$ curves in the region of high enthalpies (see Fig. 1 and table) and the slope of the current-voltage characteristics. The characteristics are the flatter, the smaller the exponent n in the conductivity equations. This is because of the reduced dependence of the conductivity on arc current.

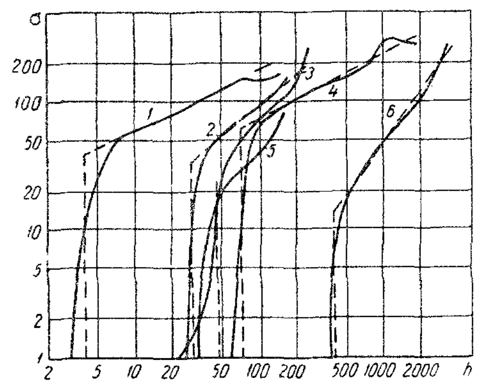

Fig. 1 Conductivity σ (mho/cm) versus enthalpy (kJ/g) at atmospheric pressure. Broken lines — approximation: 1) argon, 2) oxygen, 3) nitrogen, 4) helium, 5) lithium, 6) hydrogen.

In geometrically stabilized arcs of constant length the rising segments of the current-voltage characteristics correspond to intervals of weak dependence of σ upon h. In linear heaters with vortex stabilization increase in current is accompanied by an increase in the diameter of the arc column and a decrease in its length. Therefore in this case there is no increase in the current-voltage characteristics, however their steepness is affected by the inclination of the $\sigma = f(h)$ curves.

The current-voltage characteristics can be generalized using the values of σ_0 and h_0 presented in the table. In constructing such a generalized characteristic (Fig. 3), apart from the data of Fig. 2 (gas flow rate 4 g/sec), we also used characteristics for flowrates of 1–12 g/sec. However, the electrode diameters were the same in all the experiments. It is clear from the figure that all the points are grouped about a certain curve, the generalized characteristic of the arc heater investigated. With an accuracy sufficient for practical purposes (≈30%), the curve can be approximated by the expression

$$Ud\sigma_0/I = 3.39\,(I^2/Gd\sigma_0 h_0)^{-0.62}. \qquad (1)$$

This result indicates that in the first approximation it is possible to neglect the influence of various criteria reflecting the physical properties of the gases.

However, a closer examination of Fig. 3 shows that the curves drawn through the points corresponding to the individual gases do not coincide. For

Values of the Coefficients in the Power Approximation of the Conductivity-Enthalpy Relation (when $h < h_0$, $\sigma = 0$; when $h > h_0$, $\sigma = \sigma_0(h/h_0)^n$)

Stabilizing medium	σ_0, mho/cm	h_0, kF/g	n
Hydrogen	14	400	1.38
Helium	63	80	0.514
Lithium	13	35	1.12
Nitrogen	27	46	1.21
Air	29	44	1.19
Oxygen	34	29	0.81
Argon	40	4	0.48

example, if we draw straight lines through the points for argon and nitrogen, their inclinations will be different and they will intersect. The extreme points do not go far beyond the limits of the unified curve merely because the region of stable operation of the heater is not large. If by some means (e.g., by increasing the supply voltage) we were to expand the region of stable operation, the scatter of the points would increase. Correspondingly, there would be a reduction in the accuracy of Eq. (1). Therefore Eq. (1) is valid only for apparatus with a relatively narrow range of stable operation. (In the experiments the range of stable operation was determined by the supply voltage —480 V.)

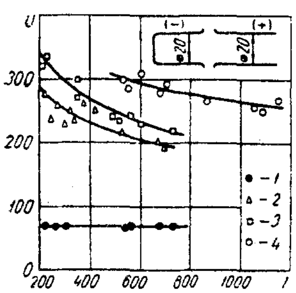

Fig. 2. Current-voltage characteristics of electric-arc heater with vortex gas stabilization: 1) argon, 2) oxygen, 3) nitrogen, 4) helium; U, V; I, A; d = 2 cm; G = 4 g/sec.

A more accurate approximating formula can be obtained by using criteria reflecting the physical properties of the gases. In the first instance, it is necessary to take into account the exponent n in the expression approximating the conductivity-enthalpy relation. However, analysis of the experimental data shows that allowing for only one of these factors does not result in any significant improvement in accuracy. It is necessary to take into account criteria that also contain other physical constants with an important effect on the processes that take place in an electric arc. The most important of these is the gas ionization potential, which is usually introduced in the form of the dimensionless number eE_i/kT. Introducing this number makes it possible to express the change in the steepness of the generalized current-voltage characteristic and hence, without loss of accuracy, remove the restrictions on the range of operating conditions. Thus, we get the approximating expression

$$Ud\sigma_0/I = an^l (eE_i/kT)^m (I^2/\sigma_0 h_0 Gd)^{-b}. \qquad (2)$$

The experimental data of Fig. 3 correspond to the following values of the coefficients: $a = 1620$; $l = -0.84$; $m = -1.0$. The exponent b also depends on n and E_i. This relation can be approximated by the expression

$$b = cn^p (eE_i/kT)^g.$$

The above experimental data are well satisfied by the following values of the coefficients: $c = 0.05$; $p = 0.34$; $g = 0.4$.

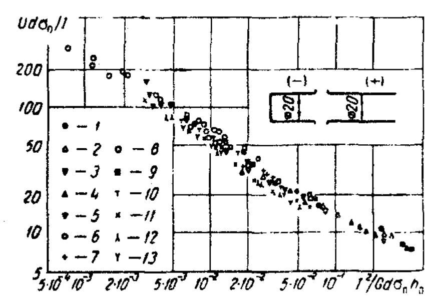

Fig. 3. Generalized current-voltage characteristic of electric-arc heater with vortex gas stabilization. Argon: 1) 4 g/sec, 2) 6, 3) 8, 4) 12; helium: 5) 1 g/sec, 6) 2, 7) 4; oxygen: 8) 4 g/sec, 9) 6, 10) 8; nitrogen: 11) 2 g/sec, 12) 44, 13) 6.

Positive values of p and g indicate that the generalized current-voltage characteristic falls the more steeply, the steeper the increase in conductivity and the greater the gas ionization potential. The effect of the sharpness of the increase in conductivity has been discussed above. The ionization potential apparently affects not only the magnitude of the conductivity but also its distribution over the section of the arc column. The negative values of the exponents l and m are due to the fact that the experimental points of the dimensionless current-voltage characteristic are grouped about a certain line, while the absolute values of $lg(I^2/Gd\sigma_0 h_0)$ are less than zero at positive values of p and g.

The error of Eq. (1) depends mainly on the accuracy of computation of the exponent b. The error can be reduced by determining b from graphs or from more accurate empirical formulas.

At present, it is still difficult to assess the final accuracy of current-voltage characteristics represented in criterial form, since we did not examine gases with high initial enthalpy or gases with low ionization potentials.

However, the result obtained gives grounds for hoping that generalized formulas suitable for practical use can be derived for broad intervals of variation of all the characteristic quantities.

NOTATION

I—current, U—voltage, G—gas flow rate, d—characteristic dimension (electrode diameter), σ_0—characteristic value of electrical conductivity, h_0—characteristic value of enthalpy, b, g, l, m, n, p—exponents, a, c—factors of proportionality, e—electronic charge, E_i—ionization potential, k—Boltzmann's constant, T—absolute temperature.

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